9MA0/01: Pure Mathematics Paper 1 Mark scheme

Question	Scheme	Marks	AOs
1 (a)	$\text{Area}(R) \approx \frac{1}{2} \times 0.5 \times \left[\underline{0.5 + 2(0.6742 + 0.8284 + 0.9686) + 1.0981}\right]$	B1	1.1b
		M1	1.1b
	$\left\{= \frac{1}{4} \times 6.5405 = 1.635125\right\} = 1.635$ (3 dp)	A1	1.1b
		(3)	
(b)	Any valid reason, for example • Increase the number of strips • Decrease the width of the strips • Use more trapezia between $x = 1$ and $x = 3$	B1	2.4
		(1)	
(c)(i)	$\left\{\int_1^3 \frac{5x}{1+\sqrt{x}}\,dx\right\} = 5(\text{"1.635"}) = 8.175$	B1ft	2.2a
(c)(ii)	$\left\{\int_1^3 \left(6 + \frac{x}{1+\sqrt{x}}\right)dx\right\} = 6(2) + (\text{"1.635"}) = 13.635$	B1ft	2.2a
		(2)	
			(6 marks)

Question 1 Notes:

(a)	
B1:	Outside brackets $\frac{1}{2} \times 0.5$ or $\frac{0.5}{2}$ or 0.25 or $\frac{1}{4}$
M1:	For structure of trapezium rule $[\ \underline{............}\ ]$.
	No errors are allowed, e.g. an omission of a *y*-ordinate or an extra *y*-ordinate or a repeated *y*-ordinate.
A1:	Correct method leading to a correct answer only of 1.635
(b)	
B1:	See scheme
(c)	
B1:	8.175 or a value which is $5 \times$ their answer to part (a)
	Note: Allow B1ft for 8.176 (to 3 dp) which is found from $5(1.63125) = 8.175625$
	Note: Do not allow an answer of 8.1886… which is found directly from integration
(d)	
B1:	13.635 or a value which is 12 + their answer to part (a)
	Note: Do not allow an answer of 13.6377… which is found directly from integration

Question	Scheme	Marks	AOs
2 (a)	$(4+5x)^{\frac{1}{2}} = (4)^{\frac{1}{2}}\left(1+\frac{5x}{4}\right)^{\frac{1}{2}} = 2\left(1+\frac{5x}{4}\right)^{\frac{1}{2}}$	B1	1.1b
	$=\{2\}\left[1+\left(\frac{1}{2}\right)\left(\frac{5x}{4}\right)+\frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2!}\left(\frac{5x}{4}\right)^2+\ldots\right]$	M1	1.1b
		A1ft	1.1b
	$= 2+\frac{5}{4}x-\frac{25}{64}x^2+\ldots$	A1	2.1
		(4)	
(b)(i)	$\left\{x=\frac{1}{10}\Rightarrow\right\}(4+5(0.1))^{\frac{1}{2}}$	M1	1.1b
	$=\sqrt{4.5} = \frac{3}{2}\sqrt{2}$ or $\frac{3}{\sqrt{2}}$		
	$\frac{3}{2}\sqrt{2}$ or $1.5\sqrt{2}$ or $\frac{3}{\sqrt{2}} = 2+\frac{5}{4}\left(\frac{1}{10}\right)-\frac{25}{64}\left(\frac{1}{10}\right)^2+\ldots\ \{=2.121\ldots\}$ $\Rightarrow \frac{3}{2}\sqrt{2}=\frac{543}{256}$ or $\frac{3}{\sqrt{2}}=\frac{543}{256} \Rightarrow \sqrt{2}=\ldots$	M1	3.1a
	So, $\sqrt{2}=\frac{181}{128}$ or $\sqrt{2}=\frac{256}{181}$	A1	1.1b
(b)(ii)	$x=\frac{1}{10}$ satisfies $\lvert x\rvert<\frac{4}{5}$ (o.e.), so the approximation is valid.	B1	2.3
		(4)	
			(8 marks)

Question 2 Notes:	
(a)	
B1:	Manipulates $(4+5x)^{\frac{1}{2}}$ by taking out a factor of $(4)^{\frac{1}{2}}$ or 2
M1:	Expands $(...+\lambda x)^{\frac{1}{2}}$ to give at least 2 terms which can be simplified or un-simplified, E.g. $1+\left(\frac{1}{2}\right)(\lambda x)$ or $\left(\frac{1}{2}\right)(\lambda x)+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}(\lambda x)^2$ or $1+...+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}(\lambda x)^2$ where λ is a numerical value and **where** $\lambda \neq 1$.
A1ft:	A correct simplified or un-simplified $1+\left(\frac{1}{2}\right)(\lambda x)+\frac{(\frac{1}{2})(-\frac{1}{2})}{2!}(\lambda x)^2$ expansion with **consistent** (λx)
A1:	Fully correct solution leading to $2+\frac{5}{4}x+kx^2$, where $k=-\frac{25}{64}$
(b)(i)	
M1:	Attempts to substitute $x=\frac{1}{10}$ or 0.1 into $(4+5x)^{\frac{1}{2}}$
M1:	A complete method of finding an approximate value for $\sqrt{2}$. E.g. • substituting $x=\frac{1}{10}$ or 0.1 into their part (a) binomial expansion and equating the result to an expression of the form $\alpha\sqrt{2}$ or $\frac{\beta}{\sqrt{2}}$; $\alpha, \beta \neq 0$ • followed by re-arranging to give $\sqrt{2}=...$
A1:	$\frac{181}{128}$ **or any equivalent fraction**, e.g. $\frac{362}{256}$ or $\frac{543}{384}$ Also allow $\frac{256}{181}$ **or any equivalent fraction**
(b)(ii)	
B1:	Explains that the approximation is valid because $x=\frac{1}{10}$ satisfies $\lvert x\rvert<\frac{4}{5}$

Question	Scheme	Marks	AOs
3 (a)	$a_1 = 3$, $a_2 = 0$, $a_3 = 1.5$, $a_4 = 3$	M1	1.1b
	$\sum_{r=1}^{100} a_r = 33(4.5) + 3$	M1	2.2a
	$= 151.5$	A1	1.1b
		(3)	
(b)	$\sum_{r=1}^{100} a_r + \sum_{r=1}^{99} a_r = (2)(151.5) - 3 = 300$	B1ft	2.2a
		(1)	
		(4 marks)	

Question 3 Notes:

(a)	
M1:	Uses the formula $a_{n+1} = \dfrac{a_n - 3}{a_n - 2}$, with $a_1 = 3$ to generate values for a_2, a_3 and a_4
M1:	Finds $a_4 = 3$ and deduces $\sum_{r=1}^{100} a_r = 33(\text{"3"} + \text{"0"} + \text{"1.5"}) + \text{"3"}$
A1:	which leads to a correct answer of 151.5
(b)	
B1ft:	Follow through on their periodic function. Deduces that either • $\sum_{r=1}^{100} a_r + \sum_{r=1}^{99} a_r = (2)(\text{"151.5"}) - 3 = 300$ • $\sum_{r=1}^{100} a_r + \sum_{r=1}^{99} a_r = \text{"151.5"} + (33)(\text{"3"} + \text{"0"} + \text{"1.5"}) = 151.5 + 148.5 = 300$

Question	Scheme	Marks	AOs
4 (a)	$\overrightarrow{OA} = \mathbf{i} + 7\mathbf{j} - 2\mathbf{k}$, $\overrightarrow{OB} = 4\mathbf{i} + 3\mathbf{j} + 3\mathbf{k}$, $\overrightarrow{OC} = 2\mathbf{i} + 10\mathbf{j} + 9\mathbf{k}$		
	$\overrightarrow{OD} = \overrightarrow{OC} + \overrightarrow{BA} = (2\mathbf{i} + 10\mathbf{j} + 9\mathbf{k}) + (-3\mathbf{i} + 4\mathbf{j} - 5\mathbf{k})$ or $\overrightarrow{OD} = \overrightarrow{OA} + \overrightarrow{BC} = (\mathbf{i} + 7\mathbf{j} - 2\mathbf{k}) + (-2\mathbf{i} + 7\mathbf{j} + 6\mathbf{k})$	M1	3.1a
	So $\overrightarrow{OD} = -\mathbf{i} + 14\mathbf{j} + 4\mathbf{k}$	A1	1.1b
		(2)	
(b)	$\left\{\overrightarrow{AB} = 3\mathbf{i} - 4\mathbf{j} + 5\mathbf{k} \Rightarrow \right\}$ $\left\lvert\overrightarrow{AB}\right\rvert = \sqrt{(3)^2 + (-4)^2 + (5)^2}$ $\left\{= \sqrt{50} = 5\sqrt{2}\right\}$	M1	1.1b
	As $\left\lvert\overrightarrow{AX}\right\rvert = 10\sqrt{2}$ then $\left\lvert\overrightarrow{AX}\right\rvert = 2\left\lvert\overrightarrow{AB}\right\rvert \Rightarrow \overrightarrow{AX} = 2\overrightarrow{AB}$		
	$\overrightarrow{OX} = \overrightarrow{OA} + 2\overrightarrow{AB} = (\mathbf{i} + 7\mathbf{j} - 2\mathbf{k}) + 2(3\mathbf{i} - 4\mathbf{j} + 5\mathbf{k})$ or $\overrightarrow{OX} = \overrightarrow{OB} + \overrightarrow{AB} = (4 + 3\mathbf{j} + 3\mathbf{k}) + (3\mathbf{i} - 4\mathbf{j} + 5\mathbf{k})$	M1	3.1a
	So $\overrightarrow{OX} = 7\mathbf{i} - \mathbf{j} + 8\mathbf{k}$ only	A1	1.1b
		(3)	
			(5 marks)

Question 4 Notes:

(a)	
M1:	A complete method for finding the position vector of D
A1:	$-\mathbf{i} + 14\mathbf{j} + 4\mathbf{k}$ or $\begin{pmatrix} -1 \\ 14 \\ 4 \end{pmatrix}$
(b)	
M1:	A complete attempt to find $\left\lvert\overrightarrow{AB}\right\rvert$ or $\left\lvert\overrightarrow{BA}\right\rvert$
M1:	A complete process for finding the position vector of X
A1:	$7\mathbf{i} - \mathbf{j} + 8\mathbf{k}$ or $\begin{pmatrix} 7 \\ -1 \\ 8 \end{pmatrix}$

Question	Scheme	Marks	AOs
5 (a)(i)	$f(x) = x^3 + ax^2 - ax + 48, \ x \in \mathbb{R}$		
	$f(-6) = (-6)^3 + a(-6)^2 - a(-6) + 48$	M1	1.1b
	$= -216 + 36a + 6a + 48 = 0 \Rightarrow 42a = 168 \Rightarrow a = 4$ *	A1*	1.1b
(a)(ii)	Hence, $f(x) = (x+6)(x^2 - 2x + 8)$	M1	2.2a
		A1	1.1b
		(4)	
(b)	$2\log_2(x+2) + \log_2 x - \log_2(x-6) = 3$		
	E.g. • $\log_2(x+2)^2 + \log_2 x - \log_2(x-6) = 3$ • $2\log_2(x+2) + \log_2\left(\frac{x}{x-6}\right) = 3$	M1	1.2
	$\log_2\left(\frac{x(x+2)^2}{(x-6)}\right) = 3$ $\quad \left[\text{or } \log_2\left(x(x+2)^2\right) = \log_2\left(8(x-6)\right)\right]$	M1	1.1b
	$\left(\frac{x(x+2)^2}{(x-6)}\right) = 2^3$ $\quad \left\{\text{i.e. } \log_2 a = 3 \Rightarrow a = 2^3 \text{ or } 8\right\}$	B1	1.1b
	$x(x+2)^2 = 8(x-6) \Rightarrow x(x^2 + 4x + 4) = 8x - 48$		
	$\Rightarrow x^3 + 4x^3 + 4x = 8x - 48 \Rightarrow x^3 + 4x^3 - 4x + 48 = 0$ *	A1 *	2.1
		(4)	
(c)	$2\log_2(x+2) + \log_2 x - \log_2(x-6) = 3 \Rightarrow x^3 + 4x^3 - 4x + 48 = 0$		
	$\Rightarrow (x+6)(x^2 - 2x + 8) = 0$		
	Reason 1: E.g. • $\log_2 x$ is not defined when $x = -6$ • $\log_2(x-6)$ is not defined when $x = -6$ • $x = -6$, but $\log_2 x$ is only defined for $x > 0$ Reason 2: • $b^2 - 4ac = -28 < 0$, so $(x^2 - 2x + 8) = 0$ has no (real) roots		
	At least one of Reason 1 or Reason 2	B1	2.4
	Both Reason 1 and Reason 2	B1	2.1
		(2)	
			(10 marks)

Question 5 Notes:	
(a)(i)	
M1:	Applies $\mathrm{f}(-6)$
A1*:	Applies $\mathrm{f}(-6) = 0$ to show that $a = 4$
(a)(ii)	
M1:	Deduces $(x + 6)$ is a factor of $\mathrm{f}(x)$ and attempts to find a quadratic factor of $\mathrm{f}(x)$ by either equating coefficients or by algebraic long division
A1:	$(x+6)(x^2 - 2x + 8)$
(b)	
M1:	Evidence of applying a correct law of logarithms
M1:	Uses correct laws of logarithms to give either • an expression of the form $\log_2(\mathrm{h}(x)) = k$, where *k* is a constant • an expression of the form $\log_2(\mathrm{g}(x)) = \log_2(\mathrm{h}(x))$
B1:	Evidence in their working of $\log_2 a = 3 \Rightarrow a = 2^3$ or 8
A1*:	Correctly proves $x^3 + 4x^3 - 4x + 48 = 0$ with no errors seen
(c)	
B1:	See scheme
B1:	See scheme

Question	Scheme	Marks	AOs
6 (a)	Attempts to use an appropriate model; e.g. $y = A(3-x)(3+x)$ or $y = A(9-x^2)$	M1	3.3
	e.g. $y = A(9-x^2)$ Substitutes $x=0, y=5 \Rightarrow 5 = A(9-0) \Rightarrow A = \frac{5}{9}$	M1	3.1b
	$y = \frac{5}{9}(9-x^2)$ or $y = \frac{5}{9}(3-x)(3+x)$, $\{-3 \leqslant x \leqslant 3\}$	A1	1.1b
		(3)	
(b)	Substitutes $x = \frac{2.4}{2}$ into their $y = \frac{5}{9}(9-x^2)$	M1	3.4
	$y = \frac{5}{9}(9-x^2) = 4.2 > 4.1 \Rightarrow$ Coach can enter the tunnel	A1	2.2b
		(2)	
(b) Alt 1	$4.1 = \frac{5}{9}(9-x^2) \Rightarrow x = \frac{9\sqrt{2}}{10}$, so maximum width $= 2\left(\frac{9\sqrt{2}}{10}\right)$	M1	3.4
	$= 2.545... > 2.4 \Rightarrow$ Coach can enter the tunnel	A1	2.2b
		(2)	
(c)	E.g. • Coach needs to enter through the centre of the tunnel. This will only be possible if it is a one-way tunnel • In real-life the road may be cambered (and not horizontal) • The quadratic curve *BCA* is modelled for the entrance to the tunnel but we do not know if this curve is valid throughout the entire length of the tunnel • There may be overhead lights in the tunnel which may block the path of the coach	B1	3.5b
		(1)	
		(6 marks)	

Question 6 Notes:

(a)	
M1:	Translates the given situation into an appropriate quadratic model – see scheme
M1:	Applies the maximum height constraint in an attempt to find the equation of the model – see scheme
A1:	Finds a suitable equation – see scheme
(b)	
M1:	See scheme
A1:	Applies a fully correct argument to infer {by assuming that curve *BCA* is quadratic and the given measurements are correct}, that is possible for the coach to enter the tunnel
(c)	
B1:	See scheme

Question	Scheme	Marks	AOs
7	$\left\{\int xe^{2x}\,dx\right\}$, $\left\{\begin{matrix} u = x & \Rightarrow \frac{du}{dx} = 1 \\ \frac{dv}{dx} = e^{2x} & \Rightarrow v = \frac{1}{2}e^{2x}\end{matrix}\right\}$		
	$\left\{\int xe^{2x}\,dx\right\} = \frac{1}{2}xe^{2x} - \int \frac{1}{2}e^{2x}\{dx\}$	M1	3.1a
	$\left\{\int 2e^{2x} - xe^{2x}\,dx\right\} = e^{2x} - \left(\frac{1}{2}xe^{2x} - \int \frac{1}{2}e^{2x}\{dx\}\right)$	M1	1.1b
	$= e^{2x} - \left(\frac{1}{2}xe^{2x} - \frac{1}{4}e^{2x}\right)$	A1	1.1b
	$\text{Area}(R) = \int_0^2 2e^{2x} - xe^{2x}\,dx = \left[\frac{5}{4}e^{2x} - \frac{1}{2}xe^{2x}\right]_0^2$	M1	2.2a
	$= \left(\frac{5}{4}e^4 - e^4\right) - \left(\frac{5}{4}e^{2(0)} - \frac{1}{2}(0)e^0\right) = \frac{1}{4}e^4 - \frac{5}{4}$	A1	2.1
		(5)	
7 **Alt 1**	$\left\{\int 2e^{2x} - xe^{2x}\,dx = \int (2-x)e^{2x}\,dx\right\}$, $\left\{\begin{matrix} u = 2-x & \Rightarrow \frac{du}{dx} = -1 \\ \frac{dv}{dx} = e^{2x} & \Rightarrow v = \frac{1}{2}e^{2x}\end{matrix}\right\}$		
	$= \frac{1}{2}(2-x)e^{2x} - \int -\frac{1}{2}e^{2x}\{dx\}$	M1 M1	3.1a 1.1b
	$= \frac{1}{2}(2-x)e^{2x} + \frac{1}{4}e^{2x}$	A1	1.1b
	$\left\{\text{Area}(R) = \int_0^2 (2-x)e^{2x}\,dx = \right\}\left[\frac{1}{2}(2-x)e^{2x} + \frac{1}{4}e^{2x}\right]_0^2$	M1	2.2a
	$= \left(0 + \frac{1}{4}e^4\right) - \left(\frac{1}{2}(2)e^0 + \frac{1}{4}e^0\right) = \frac{1}{4}e^4 - \frac{5}{4}$	A1	2.1
		(5)	
			(5 marks)

Question 7 Notes:	
M1:	Attempts to solve the problem by recognising the need to apply a method of integration by parts on either xe^{2x} or $(2-x)e^{2x}$. Allow this mark for either • $\pm xe^{2x} \to \pm\lambda xe^{2x} \pm \int \mu e^{2x}\{dx\}$ • $(2-x)e^{2x} \to \pm\lambda(2-x)e^{2x} \pm \int \mu e^{2x}\{dx\}$ where $\lambda, \mu \neq 0$ are constants.
M1:	For either • $2e^{2x} - xe^{2x} \to e^{2x} \pm \frac{1}{2}xe^{2x} \pm \int \frac{1}{2}e^{2x}\{dx\}$ • $(2-x)e^{2x} \to \pm\frac{1}{2}(2-x)e^{2x} \pm \int \frac{1}{2}e^{2x}\{dx\}$
A1:	Correct integration which can be simplified or un-simplified. E.g. • $2e^{2x} - xe^{2x} \to e^{2x} - \left(\frac{1}{2}xe^{2x} - \frac{1}{4}e^{2x}\right)$ • $2e^{2x} - xe^{2x} \to e^{2x} - \frac{1}{2}xe^{2x} + \frac{1}{4}e^{2x}$ • $2e^{2x} - xe^{2x} \to \frac{5}{4}e^{2x} - \frac{1}{2}xe^{2x}$ • $(2-x)e^{2x} \to \frac{1}{2}(2-x)e^{2x} + \frac{1}{4}e^{2x}$
M1:	Deduces that the upper limit is 2 and uses limits of 2 and 0 on their integrated function
A1:	Correct proof leading to $pe^4 + q$, where $p = \frac{1}{4},\ q = -\frac{5}{4}$

Question	Scheme	Marks	AOs
8 (a)	Total amount $= \dfrac{2100(1-(1.012)^{14})}{1-1.012}$ or $\dfrac{2100((1.012)^{14}-1)}{1.012-1}$	M1	3.1b
	$= 31806.9948\ldots = 31800$ (tonnes) (3 sf)	A1	1.1b
		(2)	
	Total Cost $= 5.15(2000(14)) + 6.45(31806.9948\ldots - (2000)(14))$	M1	3.1b
		M1	1.1b
	$= 5.15(28000) + 6.45(3806.9948\ldots) = 144200 + 24555.116\ldots$		
	$= 168755.116\ldots = £169000$ (nearest £1000)	A1	3.2a
		(3)	
		(5 marks)	

Question 8 Notes:

(a)	
M1:	Attempts to apply the correct geometric summation formula with either $n = 13$ or $n = 14$, $a = 2100$ and $r = 1.012$ (Condone $r = 1.12$)
A1:	Correct answer of 31800 (tonnes)
(b)	
M1:	Fully correct method to find the total cost
M1:	For either • $5.15(2000(14))$ $\{= 144200\}$ • $6.45(\text{"}31806.9948\ldots\text{"} - (2000)(14))$ $\{= 24555.116\ldots\}$ • $5.15(2000(13))$ $\{= 133900\}$ • $6.45(\text{"}29354.73794\ldots\text{"} - (2000)(13))$ $\{= 21638.059\ldots\}$
A1:	Correct answer of £169000 **Note:** Using rounded answer in part (a) gives 168710 which becomes £169000 (nearest £1000)

Question	Scheme	Marks	AOs
9	Gradient of chord $= \dfrac{(2(x+h)^3+5)-(2x^3+5)}{x+h-h}$	B1	1.1b
		M1	2.1
	$(x+h)^3 = x^3+3x^2h+3xh^2+h^3$	B1	1.1b
	Gradient of chord $= \dfrac{(2(x^3+3x^2h+3xh^2+h^3)+5)-(2x^3+5)}{1+h-1}$		
	$= \dfrac{2x^3+6x^2h+6xh^2+2h^3+5-2x^3-5}{1+h-1}$		
	$= \dfrac{6x^2h+6xh^2+2h^3}{h}$		
	$= 6x^2+6xh+2h^2$	A1	1.1b
	$\dfrac{\mathrm{d}y}{\mathrm{d}x} = \lim\limits_{h\to 0}\left(6x^2+6xh+2h^2\right) = 6x^2$ and so at P, $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 6(1)^2 = 6$	A1	2.2a
		(5)	
9 **Alt 1**	Let a point Q have x coordinate $1+h$, so $y_Q = 2(1+h)^3+5$	B1	1.1b
	$\left\{P(1,7), Q(1+h, 2(1+h)^3+3) \Rightarrow\right\}$		
	Gradient $PQ = \dfrac{2(1+h)^3+5-7}{1+h-1}$	M1	2.1
	$(1+h)^3 = 1+3h+3h^2+h^3$	B1	1.1b
	Gradient $PQ = \dfrac{2(1+3h+3h^2+h^3)+5-7}{1+h-1}$		
	$= \dfrac{2+6h+6h^2+2h^3+5-7}{1+h-1}$		
	$= \dfrac{6h+6h^2+2h^3}{h}$		
	$= 6+6h+2h^2$	A1	1.1b
	$\dfrac{\mathrm{d}y}{\mathrm{d}x} = \lim\limits_{h\to 0}\left(6+6h+2h^2\right) = 6$	A1	2.2a
		(5)	
			(5 marks)

Question 9 Notes:	
B1:	$2(x+h)^3+5,$ seen or implied
M1:	Begins the proof by attempting to write the gradient of the chord in terms of *x* and *h*
B1:	$(x+h)^3 \rightarrow x^3+3x^2h+3xh^2+h^3,$ by expanding brackets or by using a correct binomial expansion
M1:	Correct process to obtain the gradient of the chord as $\alpha x^2+\beta xh+\gamma h^2$, $\alpha, \beta, \gamma \neq 0$
A1:	Correctly shows that the gradient of the chord is $6x^2+6xh+2h^2$ and applies a limiting argument to deduce when $y=2x^3+5,\ \frac{\mathrm{d}y}{\mathrm{d}x}=6x^2$. E.g. $\lim_{h\to 0}\left(6x^2+6xh+2h^2\right)=6x^2$. Finally, deduces that at the point *P*, $\frac{\mathrm{d}y}{\mathrm{d}x}=6.$ **Note:** δx can be used in place of *h*
Alt 1	
B1:	Writes down the *y* coordinate of a point close to *P*. E.g. For a point *Q* with $x=1+h,\ \{y_Q\}=2(1+h)^3+5$
M1:	Begins the proof by attempting to write the gradient of the chord *PQ* in terms of *h*
B1:	$(1+h)^3 \rightarrow 1+3h+3h^2+h^3,$ by expanding brackets or by using a correct binomial expansion
M1:	Correct process to obtain the gradient of the chord *PQ* as $\alpha+\beta h+\gamma h^2$, $\alpha, \beta, \gamma \neq 0$
A1:	Correctly shows that the gradient of *PQ* is $6+6h+2h^2$ and applies a limiting argument to deduce that at the point P on $y=2x^3+5,\ \frac{\mathrm{d}y}{\mathrm{d}x}=6.$ E.g. $\lim_{h\to 0}\left(6+6h+2h^2\right)=6$ **Note:** For Alt 1, δx can be used in place of *h*

Question	Scheme	Marks	AOs
10 (a)	$y = \frac{3x-5}{x+1} \Rightarrow y(x+1) = 3x-5 \Rightarrow xy + y = 3x - 5 \Rightarrow y + 5 = 3x - xy$	M1	1.1b
	$\Rightarrow y + 5 = x(3-y) \Rightarrow \frac{y+5}{3-y} = x$	M1	2.1
	Hence $f^{-1}(x) = \frac{x+5}{3-x}, \quad x \in \mathbb{R},\ x \neq 3$	A1	2.5
		(3)	
(b)	$ff(x) = \dfrac{3\left(\frac{3x-5}{x+1}\right) - 5}{\left(\frac{3x-5}{x+1}\right) + 1}$	M1	1.1a
	$= \dfrac{\frac{3(3x-5) - 5(x+1)}{x+1}}{\frac{(3x-5) + (x+1)}{x+1}}$	M1	1.1b
		A1	1.1b
	$= \frac{9x - 15 - 5x - 5}{3x - 5 + x + 1} = \frac{4x - 20}{4x - 4} = \frac{x-5}{x-1}$ (note that $a = -5$)	A1	2.1
		(4)	
(c)	$fg(2) = f(4-6) = f(-2) = \frac{3(-2)-5}{-2+1}; = 11$	M1	1.1b
		A1	1.1b
		(2)	
(d)	$g(x) = x^2 - 3x = (x-1.5)^2 - 2.25$. Hence $g_{min} = -2.25$	M1	2.1
	Either $g_{min} = -2.25$ or $g(x) \geqslant -2.25$ or $g(5) = 25 - 15 = 10$	B1	1.1b
	$-2.25 \leqslant g(x) \leqslant 10$ or $-2.25 \leqslant y \leqslant 10$	A1	1.1b
		(3)	
(e)	E.g. • the function g is many-one • the function g is not one-one • the inverse is one-many • $g(0) = g(3) = 0$	B1	2.4
		(1)	
		(13 marks)	

Question 10 Notes:	
(a)	
M1:	Attempts to find the inverse by cross-multiplying and an attempt to collect all the *x*-terms (or swapped *y*-terms) onto one side
M1:	A fully correct method to find the inverse
A1:	A correct $f^{-1}(x) = \frac{x+5}{3-x}$, $x \in \mathbb{R}$, $x \neq 3$, expressed fully in function notation (including the domain)
(b)	
M1:	Attempts to substitute $f(x) = \frac{3x-5}{x+1}$ into $\frac{3f(x)-5}{f(x)+1}$
M1:	Applies a method of "rationalising the denominator" for both their numerator and their denominator.
A1:	$\dfrac{\frac{3(3x-5)-5(x+1)}{x+1}}{\frac{(3x-5)+(x+1)}{x+1}}$ which can be simplified or un-simplified
A1:	Shows $ff(x) = \frac{x+a}{x-1}$ where $a = -5$ or $ff(x) = \frac{x-5}{x-1}$, with no errors seen.
(c)	
M1:	Attempts to substitute the result of $g(2)$ into f
A1:	Correctly obtains $fg(2) = 11$
(d)	
M1:	Full method to establish the minimum of g. E.g. • $(x \pm \alpha)^2 + \beta$ leading to $g_{min} = \beta$ • Finds the value of *x* for which $g'(x) = 0$ and inserts this value of *x* back into $g(x)$ in order to find to g_{min}
B1:	For either • finding the correct minimum value of g (Can be implied by $g(x) \geqslant -2.25$ or $g(x) > -2.25$) • stating $g(5) = 25 - 15 = 10$
A1:	States the correct range for g. E.g. $-2.25 \leqslant g(x) \leqslant 10$ or $-2.25 \leqslant y \leqslant 10$
(e)	
B1:	See scheme

Question	Scheme	Marks	AOs
11 (a)	$f'(x) = k - 4x - 3x^2$		
	$f''(x) = -4 - 6x = 0$	M1	1.1b
	<u>Criteria 1</u> **Either** $f''(x) = -4 - 6x = 0 \Rightarrow x = \frac{4}{-6} \Rightarrow x = -\frac{2}{3}$ **or** $f''\left(-\frac{2}{3}\right) = -4 - 6\left(-\frac{2}{3}\right) = 0$ **<u>Criteria 2</u>** **Either** • $f''(-0.7) = -4 - 6(-0.7) = 0.2 > 0$ $f''(-0.6) = -4 - 6(-0.6) = -0.4 < 0$ **or** • $f'''\left(-\frac{2}{3}\right) = -6 \neq 0$		
	At least one of Criteria 1 or Criteria 2	B1	2.4
	Both Criteria 1 and Criteria 2 **and** concludes *C* has a point of inflection at $x = -\frac{2}{3}$	A1	2.1
		(3)	
(b)	$f'(x) = k - 4x - 3x^2$, $AB = 4\sqrt{2}$		
	$f(x) = kx - 2x^2 - x^3 \{+c\}$	M1	1.1b
		A1	1.1b
	$f(0) = 0$ or $(0, 0) \Rightarrow c = 0 \Rightarrow f(x) = kx - 2x^2 - x^3$ $\{f(x) = 0 \Rightarrow\}$ $f(x) = x(k - 2x - x^2) = 0 \Rightarrow \{x = 0,\}$ $k - 2x - x^2 = 0$	A1	2.2a
	$\{x^2 + 2x - k = 0\} \Rightarrow (x+1)^2 - 1 - k = 0$, $x = ...$	M1	2.1
	$\Rightarrow x = -1 \pm \sqrt{k+1}$	A1	1.1b
	$AB = \left(-1 + \sqrt{k+1}\right) - \left(-1 - \sqrt{k+1}\right) = 4\sqrt{2} \Rightarrow k = ...$	M1	2.1
	So, $2\sqrt{k+1} = 4\sqrt{2} \Rightarrow k = 7$	A1	1.1b
		(7)	
			(10 marks)

Question 11 Notes:	
(a)	
M1:	E.g. • attempts to find $\mathrm{f}''\left(-\frac{2}{3}\right)$ • finds $\mathrm{f}''(x)$ and sets the result equal to 0
B1:	See scheme
A1:	See scheme
(b)	
M1:	Integrates $\mathrm{f}'(x)$ to give $\mathrm{f}(x) = \pm kx \pm \alpha x^2 \pm \beta x^3,\ \alpha, \beta \neq 0$ with or without the constant of integration
A1:	$\mathrm{f}(x) = kx - 2x^2 - x^3$, with or without the constant of integration
A1:	Finds $\mathrm{f}(x) = kx - 2x^2 - x^3 + c$, and makes some reference to $y = \mathrm{f}(x)$ passing through the origin to deduce $c = 0$. Proceeds to produce the result $k - 2x - x^2 = 0$ or $x^2 + 2x - k = 0$
M1:	Uses a valid method to solve the quadratic equation to give *x* in terms of *k*
A1	Correct roots for *x* in terms of *k*. i.e. $x = -1 \pm \sqrt{k+1}$
M1:	Applies $AB = 4\sqrt{2}$ on $x = -1 \pm \sqrt{k+1}$ in a complete method to find $k = ...$
A1:	Finds $k = 7$ from correct solution only

Question	Scheme	Marks	AOs
12	$\int_0^{\frac{\pi}{2}} \frac{\sin 2\theta}{1+\cos\theta}\, \mathrm{d}\theta$		
	Attempts this question by applying the substitution $u = 1+\cos\theta$ and progresses as far as achieving $\int \ldots \frac{(u-1)}{u} \ldots$	M1	3.1a
	$u = 1+\cos\theta \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}\theta} = -\sin\theta$ and $\sin 2\theta = 2\sin\theta\cos\theta$	M1	1.1b
	$\left\{\int \frac{\sin 2\theta}{1+\cos\theta}\,\mathrm{d}\theta = \right\} \int \frac{2\sin\theta\cos\theta}{1+\cos\theta}\,\mathrm{d}\theta = \int \frac{-2(u-1)}{u}\,\mathrm{d}u$	A1	2.1
	$-2\int\left(1-\frac{1}{u}\right)\mathrm{d}u = -2(u - \ln u)$	M1 M1	1.1b 1.1b
	$\left\{\int_0^{\frac{\pi}{2}} \frac{\sin 2\theta}{1+\cos\theta}\,\mathrm{d}\theta = \right\} = -2\big[u - \ln u\big]_2^1 = -2((1-\ln 1)-(2-\ln 2))$	M1	1.1b
	$= -2(-1+\ln 2) = 2 - 2\ln 2$ *	A1*	2.1
		(7)	
12 **Alt 1**	Attempts this question by applying the substitution $u = \cos\theta$ and progresses as far as achieving $\int \ldots \frac{u}{u+1} \ldots$	M1	3.1a
	$u = \cos\theta \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}\theta} = -\sin\theta$ and $\sin 2\theta = 2\sin\theta\cos\theta$	M1	1.1b
	$\left\{\int \frac{\sin 2\theta}{1+\cos\theta}\,\mathrm{d}\theta = \right\} \int \frac{2\sin\theta\cos\theta}{1+\cos\theta}\,\mathrm{d}\theta = \int \frac{-2u}{u+1}\,\mathrm{d}u$	A1	2.1
	$\left\{= -2\int \frac{(u+1)-1}{u+1}\,\mathrm{d}u = -2\int 1 - \frac{1}{u+1}\,\mathrm{d}u\right\} = -2(u - \ln(u+1))$	M1 M1	1.1b 1.1b
	$\left\{\int_0^{\frac{\pi}{2}} \frac{\sin 2\theta}{1+\cos\theta}\,\mathrm{d}\theta = \right\} = -2\big[u - \ln(u+1)\big]_1^0 = -2((0-\ln 1)-(1-\ln 2))$	M1	1.1b
	$= -2(-1+\ln 2) = 2 - 2\ln 2$ *	A1*	2.1
		(7)	
			(7 marks)

Question 12 Notes:	
M1:	See scheme
M1:	Attempts to differentiate $u = 1+\cos\theta$ to give $\frac{du}{d\theta} = ...$ and applies $\sin 2\theta = 2\sin\theta\cos\theta$
A1:	Applies $u = 1+\cos\theta$ to show that the integral becomes $\int \frac{-2(u-1)}{u}\,du$
M1:	Achieves an expression in *u* that can be directly integrated (e.g. dividing each term by *u* or applying partial fractions) and integrates to give an expression in *u* of the form $\pm\lambda u \pm \mu\ln u,\ \lambda,\mu \neq 0$
M1:	For integration in *u* of the form $\pm 2(u - \ln u)$
M1:	Applies *u*-limits of 1 and 2 to an expression of the form $\pm\lambda u \pm \mu\ln u,\ \lambda,\mu \neq 0$ and subtracts either way round
A1*:	Applies *u*-limits the right way round, i.e. • $\int_2^1 \frac{-2(u-1)}{u}\,du = -2\int_2^1 \left(1-\frac{1}{u}\right)du = -2\left[u - \ln u\right]_2^1 = -2((1-\ln 1)-(2-\ln 2))$ • $\int_2^1 \frac{-2(u-1)}{u}\,du = 2\int_1^2 \left(1-\frac{1}{u}\right)du = 2\left[u - \ln u\right]_1^2 = 2((2-\ln 2)-(1-\ln 1))$ and correctly proves $\int_0^{\frac{\pi}{2}} \frac{\sin 2\theta}{1+\cos\theta}\,d\theta = 2-2\ln 2$, with no errors seen
Alt 1	
M1:	See scheme
M1:	Attempts to differentiate $u = \cos\theta$ to give $\frac{du}{d\theta} = ...$ and applies $\sin 2\theta = 2\sin\theta\cos\theta$
A1:	Applies $u = \cos\theta$ to show that the integral becomes $\int \frac{-2u}{u+1}\,du$
M1:	Achieves an expression in *u* that can be directly integrated (e.g. by applying partial fractions or a substitution $v = u+1$) and integrates to give an expression in *u* of the form $\pm\lambda u \pm \mu\ln(u+1),\ \lambda,\mu \neq 0$ or $\pm\lambda v \pm \mu\ln v,\ \lambda,\mu \neq 0$, where $v = u+1$
M1:	For integration in *u* in the form $\pm 2(u - \ln(u+1))$
M1:	Either • Applies *u*-limits of 0 and 1 to an expression of the form $\pm\lambda u \pm \mu\ln(u+1),\ \lambda,\mu \neq 0$ and subtracts either way round • Applies *v*-limits of 1 and 2 to an expression of the form $\pm\lambda v \pm \mu\ln v,\ \lambda,\mu \neq 0$, where $v = u+1$ and subtracts either way round
A1*:	Applies *u*-limits the right way round, (o.e. in v) i.e. • $\int_1^0 \frac{-2u}{u+1}\,du = -2\int_1^0 \left(1-\frac{1}{u+1}\right)du = -2\left[u - \ln(u+1)\right]_1^0 = -2((0-\ln 1)-(1-\ln 2))$ • $\int_1^0 \frac{-2u}{u+1}\,du = 2\int_0^1 \left(1-\frac{1}{u+1}\right)du = 2\left[u - \ln(u+1)\right]_0^1 = 2((1-\ln 2)-(0-\ln 1))$ and correctly proves $\int_0^{\frac{\pi}{2}} \frac{\sin 2\theta}{1+\cos\theta}\,d\theta = 2-2\ln 2$, with no errors seen

Question	Scheme	Marks	AOs
13 (a)	$R = 2.5$	B1	1.1b
	$\tan\alpha = \frac{1.5}{2}$ o.e.	M1	1.1b
	$\alpha = 0.6435$, so $2.5\sin(\theta - 0.6435)$	A1	1.1b
		(3)	
(b)	e.g. $D = 6 + 2\sin\left(\frac{4\pi(0)}{25}\right) - 1.5\cos\left(\frac{4\pi(0)}{25}\right) = 4.5\text{m}$ or $D = 6 + 2.5\sin\left(\frac{4\pi(0)}{25} - 0.6435\right) = 4.5\text{m}$	B1	3.4
		(1)	
(c)	$D_{max} = 6 + 2.5 = 8.5\text{ m}$	B1ft	3.4
		(1)	
(d)	Sets $\frac{4\pi t}{25} - \text{"}0.6435\text{"} = \frac{5\pi}{2}$ or $\frac{\pi}{2}$	M1	1.1b
	Afternoon solution $\Rightarrow \frac{4\pi t}{25} - \text{"}0.6435\text{"} = \frac{5\pi}{2} \Rightarrow t = \frac{25}{4\pi}\left(\frac{5\pi}{2} + \text{"}0.6435\text{"}\right)$	M1	3.1b
	$\Rightarrow t = 16.9052... \Rightarrow \text{Time} = 16:54$ or 4:54 pm	A1	3.2a
		(3)	
(e)(i)	• An attempt to find the depth of water at 00:00 on 19th October 2017 for at least one of either Tom's model or Jolene's model.	M1	3.4
	• At 00:00 on 19th October 2017, Tom: $D = 3.72...\text{ m}$ and Jolene: $H = 4.5\text{ m}$ and e.g. • As $4.5 \neq 3.72$ then Jolene's model is not true • Jolene's model is not continuous at 00:00 on 19th October 2017 • Jolene's model does not continue on from where Tom's model has ended	A1	3.5a
(ii)	To make the model continuous, e.g. • $H = 5.22 + 2\sin\left(\frac{4\pi x}{25}\right) - 1.5\cos\left(\frac{4\pi x}{25}\right), \quad 0 \leqslant x < 24$ • $H = 6 + 2\sin\left(\frac{4\pi(x+24)}{25}\right) - 1.5\cos\left(\frac{4\pi(x+24)}{25}\right), \quad 0 \leqslant x < 24$	B1	3.3
		(3)	
		(11 marks)	

Question	Scheme	Marks	AOs
13 (d) Alt 1	Sets $\frac{4\pi t}{25} - "0.6435" = \frac{\pi}{2}$	M1	1.1b
	$\text{Period} = 2\pi \div \left(\frac{4\pi}{25}\right) = 12.5$ Afternoon solution $\Rightarrow t = 12.5 + \frac{25}{4\pi}\left(\frac{\pi}{2} + "0.6435"\right)$	M1	3.1b
	$\Rightarrow t = 16.9052... \Rightarrow \text{Time} = 16:54$ or 4:54 pm	A1	3.2a
		(3)	

Question 13 Notes:

(a)	
B1:	$R = 2.5$ Condone $R = \sqrt{6.25}$
M1:	For either $\tan\alpha = \frac{1.5}{2}$ or $\tan\alpha = -\frac{1.5}{2}$ or $\tan\alpha = \frac{2}{1.5}$ or $\tan\alpha = -\frac{2}{1.5}$
A1:	$\alpha = \text{awrt } 0.6435$
(b)	
B1:	Uses Tom's model to find $D = 4.5\text{ (m)}$ at 00:00 on 18th October 2017
(c)	
B1ft:	Either 8.5 or follow through "6 + their R" (by using their R found in part (a))
(d)	
M1:	Realises that $D = 6 + 2\sin\left(\frac{4\pi t}{25}\right) - 1.5\cos\left(\frac{4\pi t}{25}\right) = 6 + "2.5"\sin\left(\frac{4\pi t}{25} - "0.6435"\right)$ and so maximum depth occurs when $\sin\left(\frac{4\pi t}{25} - "0.6435"\right) = 1 \Rightarrow \frac{4\pi t}{25} - "0.6435" = \frac{\pi}{2}$ or $\frac{5\pi}{2}$
M1:	Uses the model to deduce that a p.m. solution occurs when $\frac{4\pi t}{25} - "0.6435" = \frac{5\pi}{2}$ and rearranges this equation to make $t = ...$
A1:	Finds that maximum depth occurs in the afternoon at 16:54 or 4:54 pm
(d) Alt 1	
M1:	Maximum depth occurs when $\sin\left(\frac{4\pi t}{25} - "0.6435"\right) = 1 \Rightarrow \frac{4\pi t}{25} - "0.6435" = \frac{\pi}{2}$
M1:	Rearranges to make $t = ...$ and adds on the period, where $\text{period} = 2\pi \div \left(\frac{4\pi}{25}\right) \{= 12.5\}$
A1:	Finds that maximum depth occurs in the afternoon at 16:54 or 4:54 pm

Question 13 Notes Continued:	
(e)(i)	
M1:	See scheme
A1:	See scheme
	Note: Allow Special Case M1 for a candidate who just states that Jolene’s model is not continuous at 00:00 on 19th October 2017 o.e.
(e)(ii)	
B1:	Uses the information to set up a new model for *H*. (See scheme)

Question	Scheme	Marks	AOs
14	$x = 4\cos\left(t + \frac{\pi}{6}\right), \quad y = 2\sin t$		
	$x + y = 4\left(\cos t\cos\left(\frac{\pi}{6}\right) - \sin t\sin\left(\frac{\pi}{6}\right)\right) + 2\sin t$	M1 M1	3.1a 1.1b
	$x + y = 2\sqrt{3}\cos t$	A1	1.1b
	$\left(\frac{x+y}{2\sqrt{3}}\right)^2 + \left(\frac{y}{2}\right)^2 = 1$	M1	3.1a
	$\frac{(x+y)^2}{12} + \frac{y^2}{4} = 1$		
	$(x+y)^2 + 3y^2 = 12$	A1	2.1
		(5)	
14 **Alt 1**	$(x+y)^2 = \left(4\cos\left(t + \frac{\pi}{6}\right) + 2\sin t\right)^2$		
	$= \left(4\left(\cos t\cos\left(\frac{\pi}{6}\right) - \sin t\sin\left(\frac{\pi}{6}\right)\right) + 2\sin t\right)^2$	M1 M1	3.1a 1.1b
	$= \left(2\sqrt{3}\cos t\right)^2$ or $12\cos^2 t$	A1	1.1b
	So, $(x+y)^2 = 12(1 - \sin^2 t) = 12 - 12\sin^2 t = 12 - 12\left(\frac{y}{2}\right)^2$	M1	3.1a
	$(x+y)^2 + 3y^2 = 12$	A1	2.1
		(5)	
	(5 marks)		

Question 14 Notes:

M1:	Looks ahead to the final result and uses the compound angle formula in a full attempt to write down an expression for $x + y$ which is in terms of *t* only.
M1:	Applies the compound angle formula on their term in *x*. E.g. $\cos\left(t + \frac{\pi}{6}\right) \to \cos t\cos\left(\frac{\pi}{6}\right) \pm \sin t\sin\left(\frac{\pi}{6}\right)$
A1:	Uses correct algebra to find $x + y = 2\sqrt{3}\cos t$
M1:	Complete strategy of applying $\cos^2 t + \sin^2 t = 1$ on a rearranged $x + y = "2\sqrt{3}\cos t"$, $y = 2\sin t$ to achieve an equation in *x* and *y* only
A1:	Correctly proves $(x+y)^2 + ay^2 = b$ with both $a = 3, b = 12$, and no errors seen

Question 14 Notes Continued:	
Alt 1	
M1:	Apply in the same way as in the main scheme
M1:	Apply in the same way as in the main scheme
A1:	Uses correct algebra to find $(x+y)^2 = \left(2\sqrt{3}\cos t\right)^2$ or $(x+y)^2 = 12\cos^2 t$
M1:	Complete strategy of applying $\cos^2 t + \sin^2 t = 1$ on $(x+y)^2 = \left("2\sqrt{3}\cos t"\right)^2$ to achieve an equation in *x* and *y* only
A1:	Correctly proves $(x+y)^2 + ay^2 = b$ with both $a = 3, b = 12$, and no errors seen